Logical forms and substitution instances

Philosophy and Logic
Unit 2, Section 2.1
Avoiding “impossibility”

A valid deductive argument is an argument with a valid logical form.

An argument has a valid logical form if and only if no argument with that form has true premises and a false conclusion.

Two arguments have the same logical form if one can be converted to the other by lexical substitution. (One can substitute clauses for clauses, predicates for predicates, and names for names, as long as one substitutes the same for the same throughout.)
An argument has a *deductively valid logical form* if and only if no argument with the same form has true premises and a false conclusion.

- The word “impossible” has disappeared
- It gives us a plan of attack
Logical form: the plan

1. Define what a “logical form” is
2. Define what it is for two arguments to have the *same* logical form
3. Devise a test for the question “Does any argument with the *same* form have all true premises and a false conclusion?”
   - If yes: *invalid*
   - If no: *valid*
1. What is logical form?

- A schema built up from *value-bearing signs* and *logical words* (here, *connectives*):

  - If P then Q
  - Either P or Q
  - P
  - Not P
  - Therefore, Q.
  - Therefore, Q

  - *modus ponens*
  - *disjunctive syllogism*
(a) Value-bearing signs

• The letters P, Q, R, etc.
  – Call these “sentence-letters”

• These are abbreviations for statements
  – To get a different “instance” of the same form, plug in a different statement for the given letter.
  – Sometimes called “propositional variables”

• Their values: True or False.
(b) The connectives

= All the rest of the stuff in the schemas:

<table>
<thead>
<tr>
<th>Both P and Q</th>
<th>P &amp; Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Either P or Q</td>
<td>P v Q</td>
</tr>
<tr>
<td>Not P</td>
<td>~ P</td>
</tr>
<tr>
<td>If P then Q</td>
<td>P ⊃ Q</td>
</tr>
<tr>
<td>P if and only if Q</td>
<td>P ≡ Q</td>
</tr>
</tbody>
</table>

and any synonyms of these.
Some logical forms

\[ P \supset Q \]
\[ P \]
\[ \sim P \]
\[ \sim P \]

\[ \text{Q} \]
\[ \text{Q} \]

\[ \text{modus ponens} \]
\[ \text{disjunctive syllogism} \]
(b) Which connectives?

- Logical form depends on *which* words you pick as the connectives.
- We will focus specifically on *truth-functional sentential* connectives.
The innards of assertion
(b) Which connectives?

(i.) They are *sentential* connectives

They connect together two “atomic” sentences (two statements; two independent clauses) into one “molecular” sentence.

Input “P”. Input “Q”. Output: “If P then Q”.

Input “P”. Input “Q”. Output: “Both P and Q”.
The “if …then…” box

“It is sunny”

“The winds are from the south”

“If it is sunny, then the winds are from the south.”
The “both…and…” box

“It’s sunny”

“The winds are from the south”

“It’s sunny and the winds are from the south.”
The “both…and…” box

“I’m hungry” — “I am hungry and it is after 10 o’clock”

“It’s after 10 o’clock”
(b) Which connectives?

(ii.) They are truth-functional sentential connectives

– The truth value of the result is a function of (is uniquely determined by) the truth values of the atomic parts.

– If you know the truth value of “P” and the truth value of “Q”, then you know the truth value of “Both P and Q”.
The “both…and…” box

“I’m hungry”

“It’s after 10 o’clock”

“I’m hungry, and it’s after 10 o’clock”
The “both…and…” box

“I’m hungry”

“It’s after 10 o’clock”

“I’m hungry, and it’s after 10 o’clock”
The “both…and…” box

“Hello, world!”

True!

“The sun is shining.”

True!

“The sun is shining, and it’s a beautiful day.”

True!
The “both…and…” box

“I’m hungry”

“It’s after 10 o’clock”

“Both”

False!

True!

“I’m hungry, and it’s after 10 o’clock”
The “both…and…’’box

“I’m hungry”

“It’s after 10 o’clock”

True!

False!

“I’m hungry, and it’s after 10 o’clock”

False!
The “either...or...” box

“I’m hungry”

“It’s after 10 o’clock”

“Either I’m hungry, or it’s after 10 o’clock”
The “either…or…” box

- "I’m hungry"
- "It’s after 10 o’clock"
- "Either I’m hungry, or it’s after 10 o’clock"

Truth values:
- False
- True
The “either…or…” box

“I’m hungry” - False

“It’s after 10 o’clock” - True

“Either I’m hungry, or it’s after 10 o’clock” - True!
Truth-functional sentential connectives

• What makes them “truth-functional”: the truth value of the result is *uniquely determined* by the truth values of the atomic components.

• This is true of all five kinds:
  
  Both P and Q \hspace{1cm} P \& Q  
  Either P or Q \hspace{1cm} P \lor Q  
  Not P \hspace{1cm} \sim P  
  If P then Q \hspace{1cm} P \supset Q  
  P if and only if Q \hspace{1cm} P \equiv Q
2. Having the *same* logical form

- Two arguments have the *same* logical form if one can be converted to the other by *lexical substitution*
  - Think of this as a *cut and paste* operation, with restrictions
  - You can only cut and paste independent clauses (statements. Parts that *could* have truth values.)
  - You must replace the *same* with the *same* throughout
If Connecticut borders Massachusetts, then Connecticut sometimes gets Nor’Easters.
Connecticut borders Massachusetts.
-----------------------------------------
Connecticut sometimes gets Nor’Easters.

If the celestial spheres are centered on the earth, then the sun revolves around the earth.
The celestial spheres are centered on the earth.
---------------------------------------------
The sun revolves around the earth.

If P then Q
P
Therefore Q
Same logical form (cont)

• Substitute independent clauses for independent clauses
  – (e.g., statements for statements)

• The restriction: you must substitute the same for the same, throughout
  – So the “cut and paste” must be global
Forms and instances

• The logical form is the *schema*. Sentence-letters stand in for sentences.

• The particular argument is called a *substitution instance* of that form.
  – It is what you get if you replace the sentence-letters with sentences.
  – What links the two: a “symbolization key”
By way of “symbolization keys”

A mapping from sentence-letters to sentences:

P: I am hungry
Q: It is after 10 o’clock

P ⊃ Q: If I am hungry, then it is after 10 o’clock.
P & ~Q: I am hungry, and it is not after 10 o’clock.
The wrinkle: a sentence-letter can stand for any independent clause, including molecular ones.

So these are also allowed:

P: If I am hungry, then it is after 10 o’clock.

Q: I am hungry, and it is not after 10 o’clock.

R: If I am hungry, then today is a class day and I didn’t have my afternoon snack.
Sentences vs. forms

We tax and we spend. \( p \land q \)
We tax and we spend. \( p \)
We do not spend. \( q \)
We tax and we tax. \( p \land q \)
We spend and we spend. \( q \land q \)

We tax and we spend. \( p \land p \)
We spend. \( p \land q \)
Some simple guidelines

– Avoid having a letter stand for a compound. (It’s legal, but odd).
– Make every negation explicit.
– For a given atom, always use the same letter.

• The suggestion: One letter per atom. One atom per letter.
Two kinds of exercises

1. The sentence “if the plan is not approved, then we won’t build a stadium” is a substitution instance of which of the following statement forms? Circle all that apply. For each form that fits, write out a symbolization key showing how the sentence can be derived from the given form. (5 each)

   • (a) \( \sim p \supset \sim q \)
     (b) \( p \lor q \)
     (c) \( p \land \sim q \)
     (d) \( p \supset q \)
     (e) \( p \supset \sim q \)
“if the plan is not approved, then we won’t build a stadium”

(a) \( \sim p \supset \sim q \)        (b) \( p \lor q \)                  (c) \( p \land \sim q \)
(d) \( p \supset q \)                (e) \( p \supset \sim q \)

Test each form in turn. Is there any way to write a key which will produce the sentence in question from that form? Check your answer by plugging your key into the form, and reading the result.

Look at the connectives, particularly the main connective. Watch out for negations!
“if the plan is not approved, then we won’t build a stadium”

(a) \( \sim p \supset \sim q \)  
(b) \( p \lor q \)  
(c) \( p \land \sim q \)  
(d) \( p \supset q \)  
(e) \( p \supset \sim q \)  

(a) \( p \): the plan is approved  
\( q \): we will build a stadium  
(d) \( p \): the plan is not approved  
\( q \): we won’t build a stadium  
(e) \( p \): the plan is not approved  
\( q \): we will build a stadium
Exercise 2

2. Write out three statement forms of which the following sentence is a substitution instance: “either the plan is approved and we build a stadium, or we won’t join Division I.”

- Here you need to generate the form, not the key.
- One answer that always works: “p”
- Otherwise, again watch the connectives. First find the main connective.
“either the plan is approved and we build a stadium, or we won’t join Division I.”

• p
  (p: either the plan is approved and we build a stadium, or we won’t join Division I.)

• p v q
  (p: the plan is approved and we build a stadium
   q: we won’t join Division I.)

• p v ~q
  (p: the plan is approved and we build a stadium
   q: we will join Division I.)

• (p & q) v ~r
  (p: the plan is approved
   q: we build a stadium
   r: we will join Division I.)